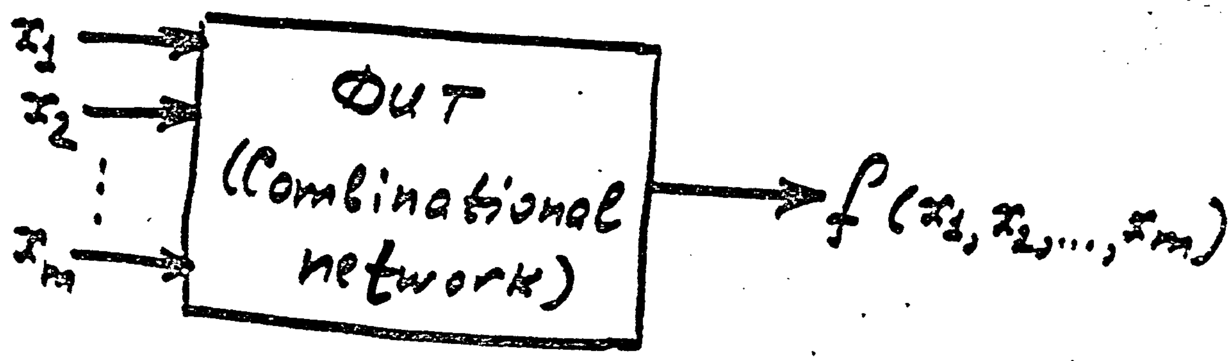
Boolean Differences

(Boolean Derivatives)

Efficient analytical technique for test generation for SSFs

Problem: for a given fault construct all test patterns detecting



Definition:

$$\boxed{\frac{\partial f}{\partial x_1}(x_2, \dots, x_m) = f(0, x_2, \dots, x_m) \oplus f(1, x_2, \dots, x_m)}$$

Testing by Boolean Difference

Theorem: (i) Input pattern $(t_1, \ldots, t_m)$ detects $x_1/0$ iff

$$\boxed{t_1 \frac{\partial f}{\partial x_1}(t_2, \ldots, t_m) = 1}$$

(ii) $(t_1, \ldots, t_m)$ detects $x_1/1$ iff

$$\boxed{\bar{t}_1 \frac{\partial f}{\partial x_1}(t_2, \ldots, t_m) = 1}$$

Test generation for detection of $x_i/0$ and $x_i/1$ reduces to computation of $\frac{\partial f}{\partial x_i}$

T3

$x_i/0$ and $x_i/1$ are undetectable iff

$$\frac{\partial f}{\partial x_i} = 0$$

Example 1

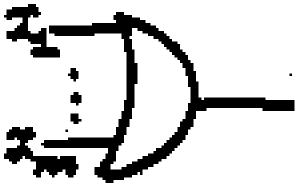
$f(x_1, x_2, \dots, x_m) = x_1 \cdot x_2 \cdot x_3 \dots x_m$

$$\frac{\partial f}{\partial x_1} = 0 x_2 \cdots x_m \oplus 1 \cdot x_2 \cdot \ldots \cdot x_m = x_2 x_3 \cdots x_m$$

Fault	Condition for test	test pattern
$x_1/0$	$x_1 x_2 x_3 \cdots x_m = 1$	$1 1 1 \dots 1$
$x_1/1$	$\overline{x}_1 x_2 x_3 \dots x_m = 1$	$0 1 1 \dots 1$
$x_2/0$	$x_1 x_2 x_3 \dots x_m = 1$	$1 1 1 \dots 1$
$x_2/1$	$x_1 \overline{x}_2 x_3 \dots x_m = 1$	$1 0 1 \dots 1$
$\vdots$		
$x_m/0$	$x_1 x_2 x_3 \dots x_m = 1$	$1 1 1 \dots 1$
$x_m/1$	$x_1 x_2 x_3 \dots \overline{x}_m = 1$	$1 1 1 \dots 0$

Optimal test:

$$\left.\begin{matrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & 0 & 1 & \dots & 1 & 1 \\ 1 & 1 & 0 & \dots & 1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & 1 & \dots & 0 & 1 \\ 1 & 1 & 1 & \dots & 1 & 0 \\ 1 & 1 & 1 & \dots & 1 & 1 \end{matrix}\right\} \; m+1 \text{ test pattern}$$

Example 2 

$f(x_1, \dots, x_m) = x_1 \vee \dots \vee x_m$ ok

$$\frac{\partial f}{\partial x_i} = (x_1 \vee \dots \vee x_{i-1} \vee 0 \vee x_{i+1} \vee \dots \vee x_m) \oplus$$

$$(x_1 \vee \dots \vee x_{i-1} \vee 1 \vee x_{i+1} \vee \dots \vee x_m) =$$

$$= (x_1 \vee \dots \vee x_{i-1} \vee x_{i+1} \vee \dots \vee x_m) \oplus 1 =$$

$$= \bar{x}_1 \dots \bar{x}_{i-1} \bar{x}_{i+1} \dots \bar{x}_m$$

Optimal test:

$$\left.\begin{matrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot & \cdot \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{matrix}\right\} \quad m+1 \text{ test pattern}$$

Example 3

$f(x_1,\dots,x_m) = x_1 \oplus \dots \oplus x_m$

$$\frac{\partial f}{\partial x_i} = (x_1 \oplus \dots \oplus x_{i-1} \oplus 0 \oplus x_{i+1} \dots \oplus x_m) \oplus$$

$$(x_1 \oplus \dots \oplus x_{i-1} \oplus 1 \oplus x_{i+1} \dots \oplus x_m) = 1$$

Optimal test $\quad 00\dots00$
$\quad 11\dots11$

detects all single input stuck-at fault for any m

XOR gates have better testability than AND and OR gates

Properties of Boolean Differences

(Rules of Algebraic Manipulations for computing $\frac{\partial f}{\partial x_i}$

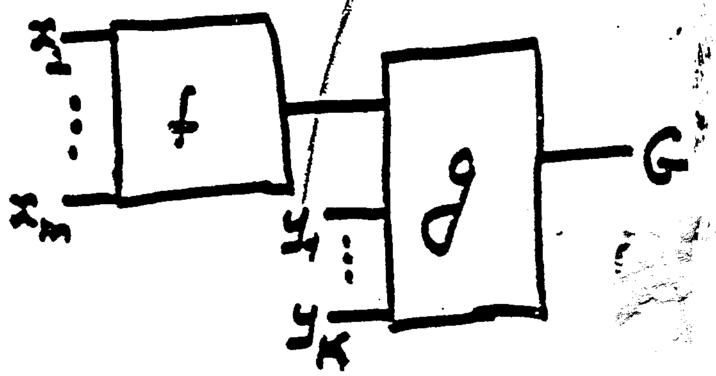
1. NOT $\quad \frac{\partial \bar{f}}{\partial x_i} = \frac{\partial f}{\partial x_i} \quad$ (e.g. $\frac{\partial (f \vee g)}{\partial x_i} = \frac{\partial (\bar{f}\,\bar{g})}{\partial x_i}$)

2. AND
$$\frac{\partial (f \cdot g)}{\partial x_i} = f \frac{\partial g}{\partial x_i} \oplus g \frac{\partial f}{\partial x_i} \oplus \frac{\partial f}{\partial x_i} \frac{\partial g}{\partial x_i}$$

3. OR
$$\frac{\partial (f \vee g)}{\partial x_i} = \bar{f} \frac{\partial g}{\partial x_i} \oplus \bar{g} \frac{\partial f}{\partial x_i} \oplus \frac{\partial f}{\partial x_i} \frac{\partial g}{\partial x_i}$$

4. XOR
$$\frac{\partial (f \oplus g)}{\partial x_i} = \frac{\partial f}{\partial x_i} \oplus \frac{\partial g}{\partial x_i}$$

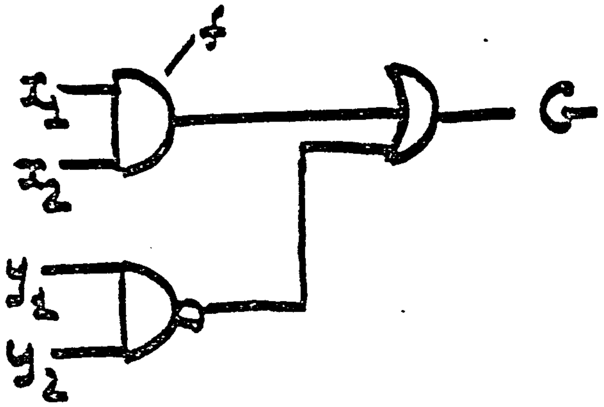
5. Simple Chain Rule



$x_i \neq y_j$

$$\frac{\partial G}{\partial x_i} = \frac{\partial g}{\partial f} \cdot \frac{\partial f}{\partial x_i}$$

Example



$f = x_1 x_2$

$g = f \vee \overline{y_1 y_2}$

$$\frac{\partial g}{\partial f} = \overline{y_1 y_2} \oplus 1 = y_1 y_2 \quad , \quad \frac{\partial f}{\partial x_1} = x_2 \quad \Rightarrow$$

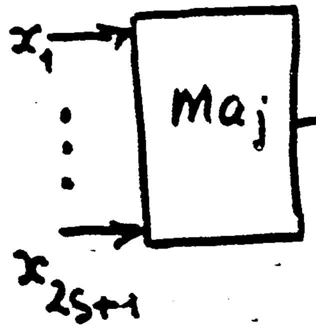
$$\frac{\partial G}{\partial x_1} = y_1 y_2 x_2$$

	$X_1 X_2$
$X_1/0$	1111
$X_1/1$	0111

y_1 y_2

Example

MAJORITY GATE



$$f(x_1, \dots, x_{2s+1}) = \begin{cases} 1, & \sum_{i=1}^{2s+1} x_i \geq s+1 \\ 0, & \sum_i x_i < s+1 \end{cases}$$

Let $T_r(m) = T_r(x_1, \dots, x_m) = \begin{cases} 1, & \sum_{i=1}^{m} x_i \geq r \\ 0, & \sum_i x_i < r \end{cases}$

$$Maj(x_1, \dots, x_{2s+1}) = T_{s+1}(2s+1)$$

$$\frac{\partial maj}{\partial x_1} = Maj(0, x_2, \dots, x_{2s+1}) \oplus Maj(1, x_2, \dots, x_{2s+1}) =$$

$$= T_{s+1}(x_2, \dots, x_{2s+1}) \oplus T_s(x_2, \dots, x_{2s+1})$$

$$= T_{s+1}(2s) \oplus T_s(2s) \Rightarrow$$

$$\frac{\partial maj}{\partial x_1} = 1 \Leftrightarrow \sum_{i=2}^{2s+1} x_i = s$$

$$\frac{\partial maj}{\partial x_i} = 1 \iff \sum_{j \neq i} x_j = s$$

Test patterns:

Faults detected

$\overbrace{0\,0 \ldots 0}^{s}\ \overbrace{1\,1\,1 \ldots 1}^{s+1}$ $\quad x_i/0 \quad i = s+1, \ldots, 2s+1$

$\overbrace{1\,1 \ldots 1\,1}^{s+1}\ \overbrace{0\,0 \ldots 0}^{s}$ $\quad x_i/0 \quad i = 1, \ldots, s$

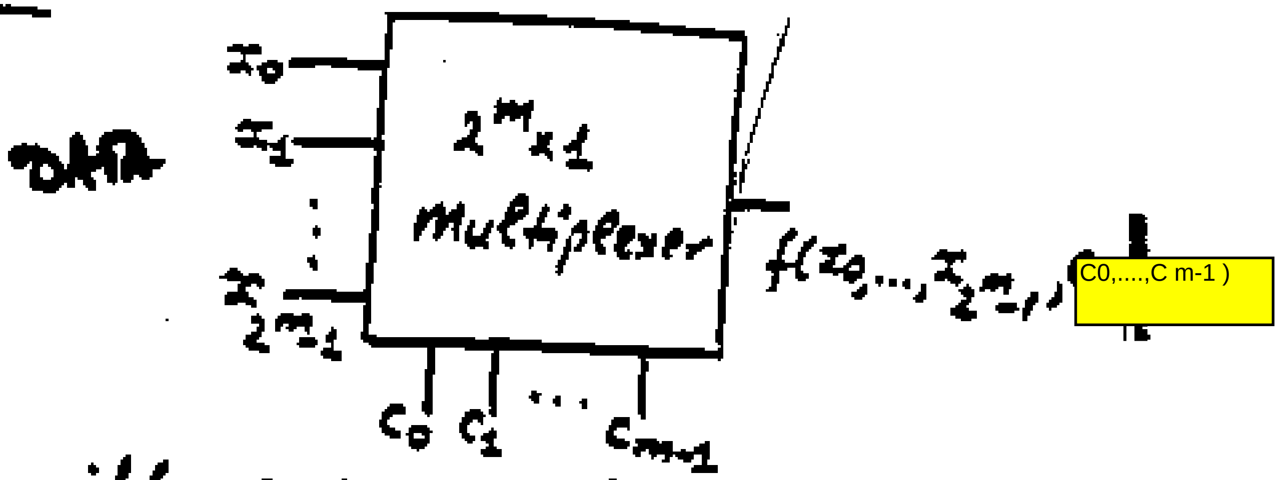
$\overbrace{0\,0 \ldots 0\,0}^{s+1}\ \overbrace{1\,1 \ldots 1}^{s}$ $\quad x_i/1 \quad i = 1, \ldots, s$

$\overbrace{1\,1 \ldots 1}^{s}\ \overbrace{0\,0\,0 \ldots 0}^{s+1}$ $\quad x_i/1 \quad i = s+1, \ldots, 2s$

Constant Test Complexity

4 test patterns for any $n = 2s+$

Example 4



$f = x_i$ iff $c = i$ CONTROL

$$\frac{\partial f}{\partial x_0} = \begin{cases} 1, & c = (0, 0, \ldots, 0); \\ 0, & \text{otherwise}; \end{cases}$$

$$\frac{\partial f}{\partial x_i} = \begin{cases} 1, & c = (c_0, \ldots, c_{m-1}) = i; \\ 0, & c \neq i; \end{cases}$$

Optimal test:

$2 \cdot 2^m$ test patterns

detects all I/O SSFs.

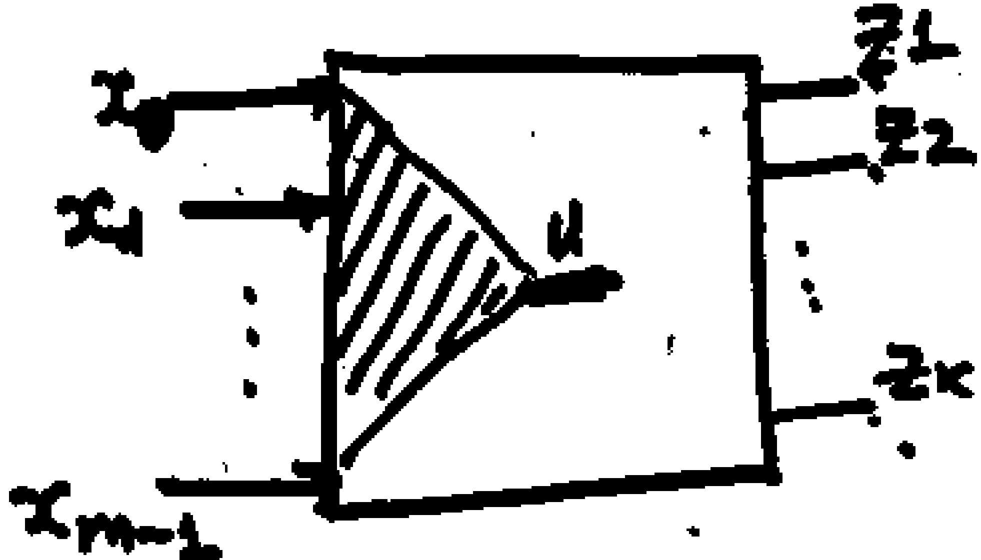
x_0	x_1	...	x_{2^m-1}	c_0	c_1	...	c_{m-1}	
0	0	...	0	0	0	...	0	$x_0/1$
1	0	...	0	0	0	...	0	$x_0/0$, $c_i/1$
0	0	...	0	0	0	...	1	$x_1/1$
0	1	...	0	0	0	...	1	$x_1/0$
...	...	...	...	...	...	...	...	...
0	0	...	0	1	1	...	1	$x_{2^m-1}/1$
0	0	...	1	1	1	...	1	$x_{2^m-1}/0$

$c_i/0$

DETECTION OF STUCK-AT FAULTS (SSFS) at INTERNAL LINES FOR DEVICES WITH MULTIPLE outputs by Boolean DIFFERENCES.

u/0 - ?

u/1 - ?



FOR DETECTION OF U/O $\Longleftrightarrow$

1) $u(x_0, x_1, \ldots, x_{m-1}) = 1$ CONTROL.

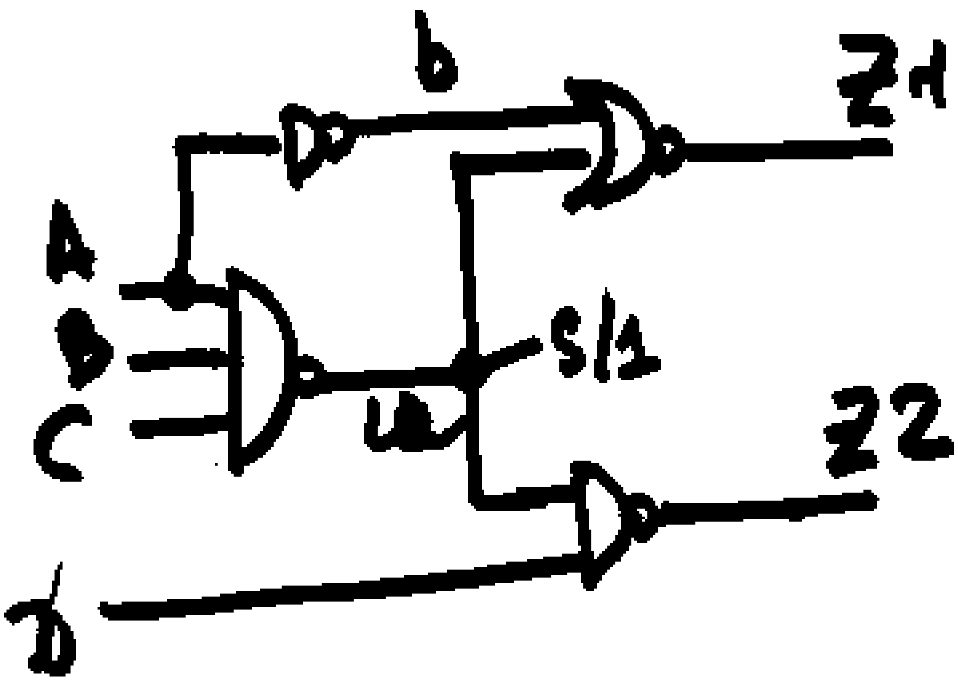
2) $\dfrac{\partial z_1}{\partial u} = 1$ or $\dfrac{\partial z_2}{\partial u} = 1 \ldots$ or $\dfrac{\partial z_k}{\partial u} = 1$

Thus

CONTROL

OBSERVE

$$\boxed{u(x_0, x_1, \ldots, x_m) \cdot \left(\frac{\partial z_1}{\partial u} \vee \frac{\partial z_2}{\partial u} \vee \ldots \vee \frac{\partial z_k}{\partial u} \right) = 1}$$



u/1 - ?

CONTROLLABILITY

1) $u = 0 \Rightarrow$

$u = \overline{ABC} = 0$

$ABC = 1 \Rightarrow A = 1, B = 1, C = 1$

Observability

$$\frac{\partial Z1}{\partial u} = \overline{(0 \vee b)} \oplus \overline{(1 \vee b)} =$$

$$= \bar{b} \oplus 0 = \bar{b}$$

$$\frac{\partial Z2}{\partial u} = (\overline{0 \cdot D}) \oplus (\overline{1 \cdot D}) = 1 \oplus \overline{D} = D.$$

$$u(A,B,C) \cdot \left(\frac{\partial Z1}{\partial u} \vee \frac{\partial Z2}{\partial u} \right) =$$

$$= ABC\,(\overline{b} \vee D) = ABC(A \vee D) =$$

$$= ABC \vee ABCD = ABC$$

TEST	A	B	C	D
	1	1	1	*

FAULTS AT
LINE u are undetectable
(CANNOT OBSERVE distortion of a
SIGNAL AT u) $\Longleftrightarrow$

any i $\quad \dfrac{\partial z_i}{\partial u} \equiv 0$