ROBUST COMPRESSION OF TEST RESPONSES

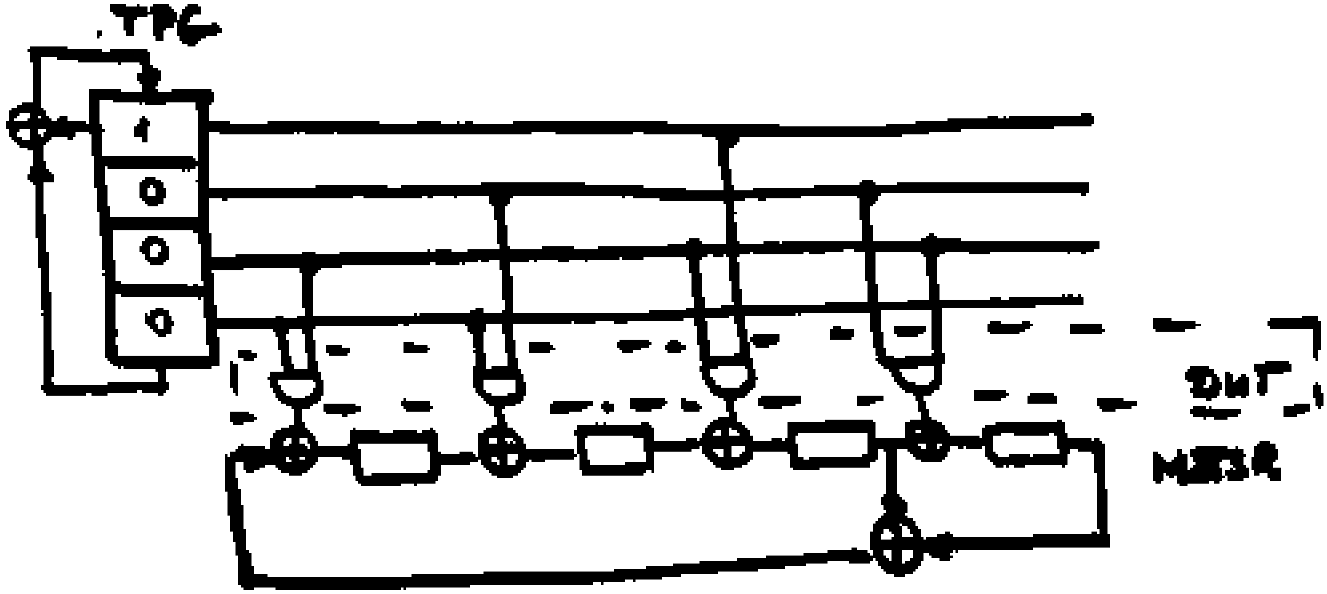
GOAL: CONSTRUCT A COMPRESSOR SUCH THAT ALIASING PROB. does not depend on a distribution of errors

SOLUTION: QUADRATIC COMPRESSORS

FOR ANY ERROR pattern probability of masking this error pattern is 2^{-r} where r is a number of bits in the quadratic signature.

Linear compressors (SIGNATURE ANALYZERS, MISRS, LFSRs) are not roburst: their performances (aliasing) depend on a device-under-test (DUT) and a selected test

EXAMPLE



- M=4. INITIAL STATE OF TEST PATTERN GENERATOR (TPG) = 1000
- TEST IS EXHAUSTIVE (EXCEPT FOR 0000)
- All (!) SINGLE and all multiple stuck-at faults are detected but MASKED IN THE MISR.

LINEAR COMPRESSORS ARE <u>NOT</u> ROBUST:

BUILT FROM FFs + XORs

H - data compression matrix

S - signature

$$S = HZ$$

$$\hat{S} = H\hat{Z}$$

$S = \hat{S}$ - ALIASING

$$HZ = H\hat{Z}$$

$$H(Z \oplus \hat{Z}) = 0$$

$He = 0$

kernel K

Prob of detection $\in \{0,1\}$ FOR A <u>GIVEN</u> error.

Let z is a fault-free response (n-bits)

$\tilde{z} = z \oplus e$ faulty response

e - error (n-bits)

$S = f(\tilde{z}) = f(z \oplus e)$ - SIGNATURE

ALIASING OF e

$$f(z) = f(z \oplus e)$$

Prob of masking e:

$$p(e) = 2^{-n} \left| z : f(z) = f(z \oplus e) \right|$$

The compressor is <u>robust</u> iff

$p(e) = 2^{-r}$ FOR <u>EVERY</u> e

r is THE SIZE (number of bits in the signature)

If THE COMPRESSOR is **LINEAR** (BUILT BY XOR GATES and FFs, e.g. LFSRs)

$$f(z \oplus e) = f(z) \oplus f(e)$$

AND

$$f(z) = f(z \oplus e) \Leftrightarrow f(e) = 0$$

THUS, FOR LINEAR COMPRESSORS

$P(e)$ is either 0 or 1,

AND ALL **LINEAR COMPRESSORS ARE NOT ROBUST**

FOR ANY LFSR IT IS POSSIBLE TO CONSTRUCT A CUT such that even for **exhaustive** testing of this CUT and compressing test responses by the LFSR will result in a low fault coverage FOR SSFs.

EXAMPLE $r=3$, QUADRATIC COMPRESSORS 14

Let output stream for a DUT is 101 011 110 010 101 011

TEST RESPONSE.

1. Select a primitive polynomial of degree r : $x^3+x+1 = P(x)$

2. PARTITION THE OUTPUT STREAM INTO DISJOINT SUBSTREAMS OF LENGTH r

$T = 2rq$, $r=3$, $q=3$

$S_1 = 101$, $S_2 = 011$

$S_3 = 110$, $S_4 = 010$

$S_5 = 101$, $S_6 = 011$

3. MULTIPLY ANY TWO STRINGS S_{2i+1} and S_{2i} as POLYNOMIALS MODULO $P(x)$ $(x^3 = x+1)$

$$S_1 \cdot S_2 = (101)(011) = (1+x^2)(1+x) =$$

$$= 1+x^2+x+x^3 = 1+x+x^2+x+1 = x^2 = (100)$$

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$S_3 \cdot S_4 = (110)(010) = (x^2+x)\, x =$

$= x^3 + x^2 = x^2 + x + 1 = (111)$

$S_5 \cdot S_6 = (101)(011) = (x^2+1)(x+1) = 100$

4. ADD UP RESULTS OF MULTIPLI-
CATIONS TO FORM r-BIT
SIGNATURE

$S = S_1 S_2 + S_3 S_4 + S_5 \cdot S_6 =$

$= 100 + 111 + 100 = 111.$

quadratic
signature.

$L = S_1 \oplus S_2 \oplus S_3 \oplus S_4 \oplus S_5 \oplus S_6 \longrightarrow$ worst <u>LINEAR</u>

<u>NOT</u> ROBUST

$QS_1 = S_1 \cdot S_2 \oplus S_3 \cdot S_4 \oplus S_5 \cdot S_6$

$QS_2 = S_1 \cdot S_3 \oplus S_2 \cdot S_4 \oplus S_5 \cdot S_6$

} – ROBUST. $P_{AL} = 2^{-r}$

<u>~~QUADRATIC~~</u> <u>QUADRATIC</u>

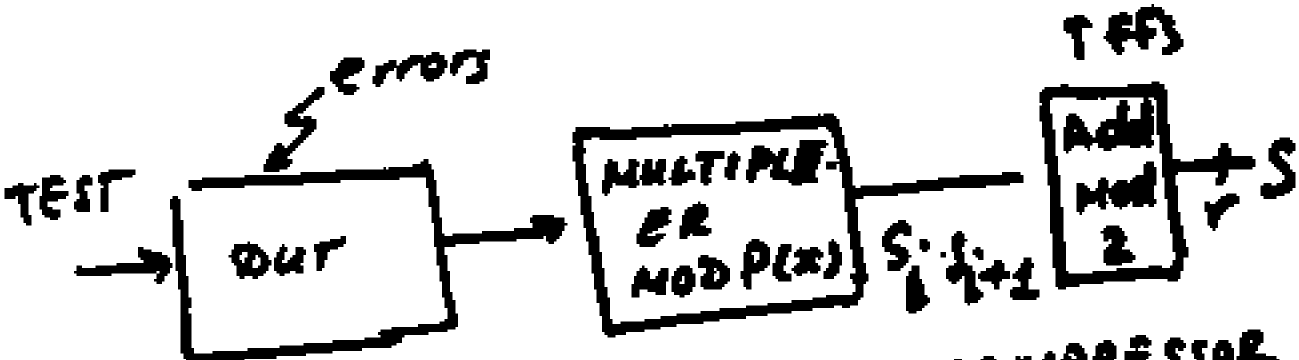
$CS = S_1 S_2 S_3 \oplus S_4 S_5 S_6$

. . . .

} <u>NOT</u> ROBUST

<u>QUBIC</u>

HARDWARE IMPLEMENTATION OF QUADRATIC COMPRESSORS



QUADRATIC COMPRESSOR

FINITE FIELD MULTIPLIER MOD P(x)

p(x) - PRIMITIVE

$$q(x) = a(x)\, b(x) \pmod{p(x)}$$

$$a(x) = a_{r-1} x^{r-1} + a_{r-2} x^{r-2} + \ldots + a_1 x + a_0$$

$$b(x) = b_{r-1} x^{r-1} + b_{r-2} x^{r-2} + \ldots + b_1 x + b_0$$

$$q(x) = q_{r-1} x^{r-1} + q_{r-2} x^{r-2} + \ldots + q_1 x + q_0$$

$$a_i, b_i, q_i \in \{0,1\}$$

FINITE FIELD MULTIPLIERS

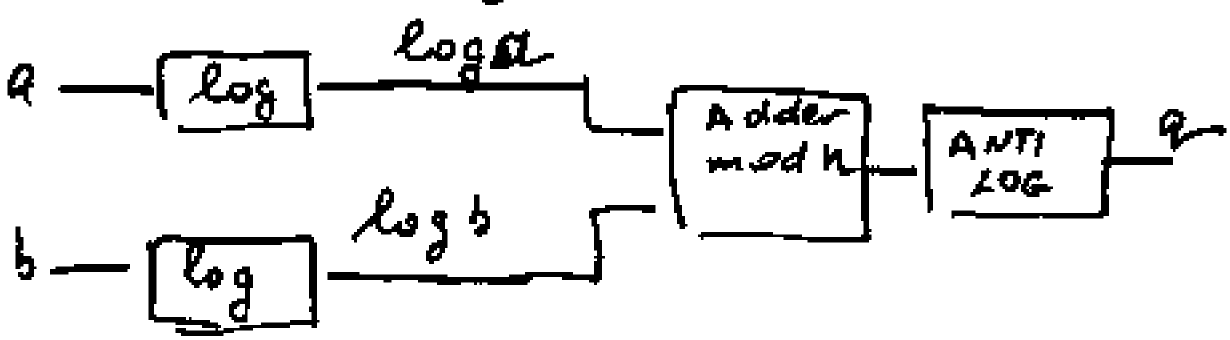
$$\underbrace{(a_0, a_1, \ldots, a_{r-1})}_{a} \; \underbrace{(b_0, b_1, \ldots, b_{r-1})}_{b} = \overbrace{(q_0, q_1, \ldots, q_{r-1})}^{q} \mod p(x)$$

$a_i, b_i, q_i \in \{0,1\}$

$\deg p(x) = r$

$n = 2^r - 1$

1. Multiplication by taking log



Example $\quad p = x^4 + x + 1 \qquad r = 4 \quad n = 15$

$a = 0101 \rightarrow \log a = 8$

$b = 1011 \rightarrow \log b = 7$

$\rightarrow \log q = 15 \overset{\text{mod } 15}{=} 0$

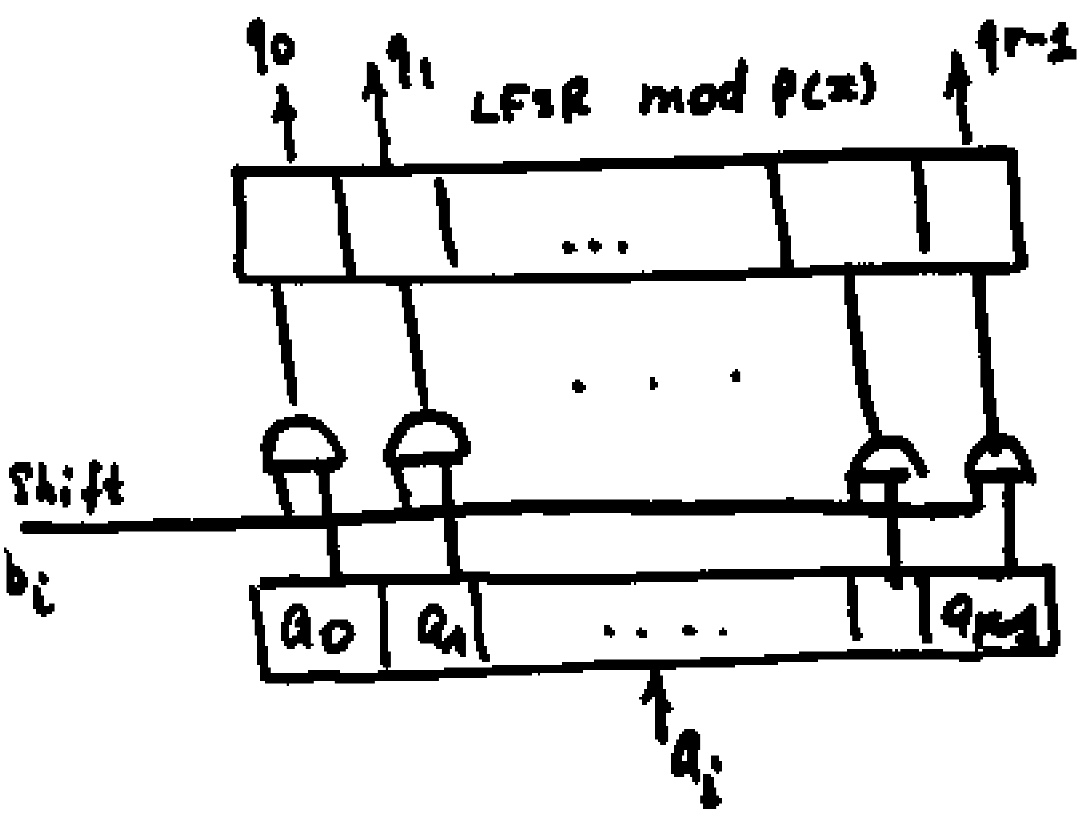
$\downarrow$

antilog $0 = 1$

$\downarrow$

$q = 0001$.

2) SERIAL MULTIPLIER MOD P(x) 24



r-STEP MULTIPLICATION

3) PARALLEL MULTIPLIER MOD $P(x)$

EXAMPLE $r=3$ $P(x)=x^3+x+1$

$q(x)=(a_2x^2+a_1x+a_0)(b_2x^2+b_1x+b_0)=$

$a_2b_2x^4+(a_1b_2+a_2b_1)x^3+(a_0b_2+a_1b_1+a_2b_0)\cdot$

$\cdot x^2+(a_0b_1+a_1b_0)x+a_0b_0=$

$=c_4x^4+c_3x^3+c_2x^2+c_1x+c_0$

Since $x^3=x+1$ and $x^4=x^2+x \Rightarrow$

$q(x)=q_2x^2+q_1x+q_0=(c_4+c_2)x^2+$

$(c_4+c_3+c_1)x+(c_3+c_0)$ mod $p(x)$

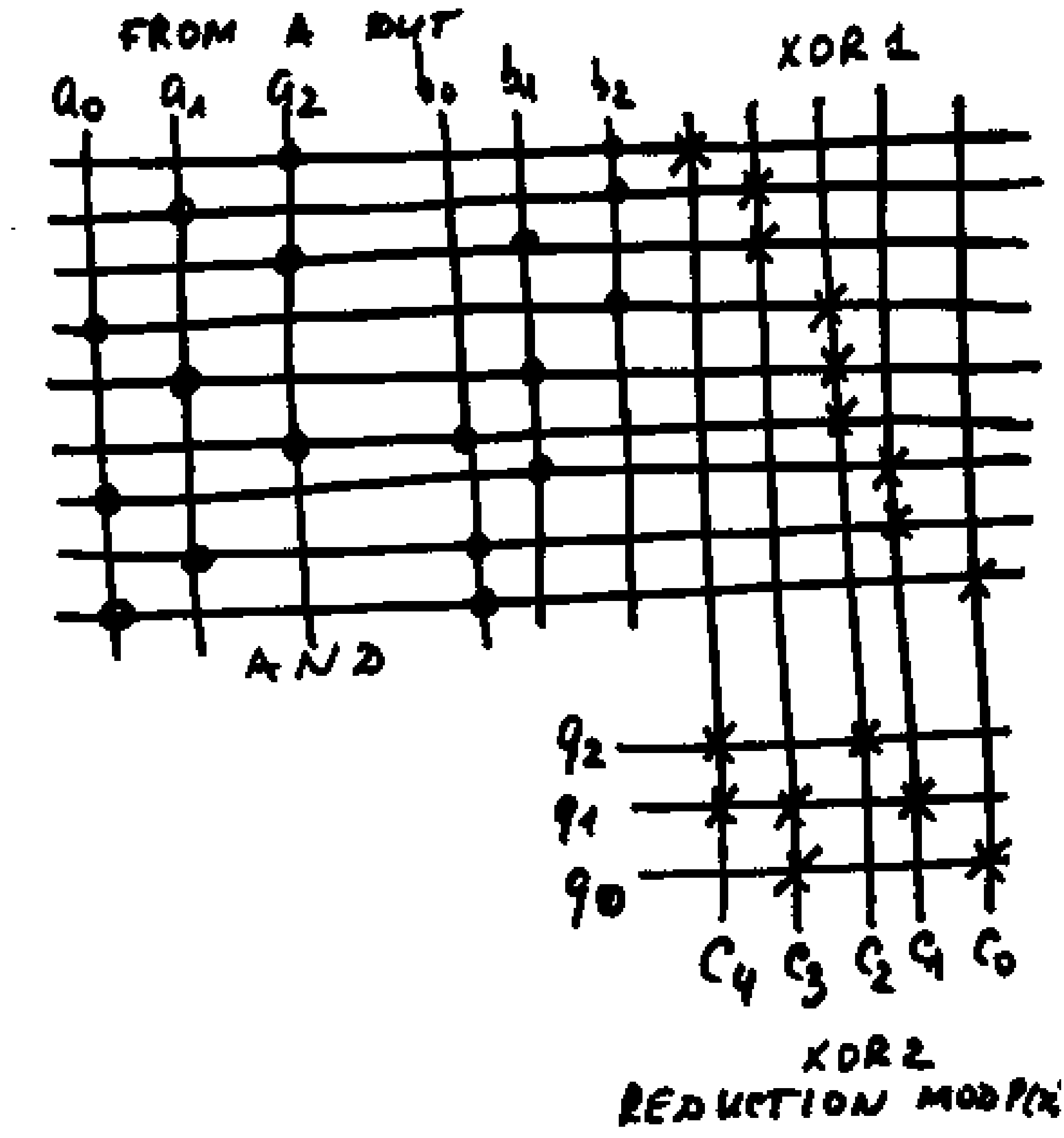
Thus

$q_2=a_2b_2+a_0b_2+a_1b_1+a_2b_0$

$q_1=a_2b_2+a_1b_2+a_2b_1+a_0b_1+a_1b_0$

$q_0=a_1b_2+a_2b_1+a_0b_0$

IMPLEMENTATION OF A PARALLEL MULTIPLIER MOD $P(x) = x^3 + x + 1$



RESULTS

1. QUADRATIC COMPRESSORS ARE ROBUST $\Rightarrow$ ALIASING PROBABILITY is equal 2^{-r} FOR an r-bit quadratic signature for any distribution of errors (assuming THAT FAULT-FREE RESPONSES ARE UNIFORMLY distributed)

. QUADRATIC COMPRESSORS require MORE HARDWARE THAN SIGNATURE ANALYSIS (TWO r-bit LFSRS AND r two-input GATES)

M. G. KARPOVSKY, P. NAGVAJARA "OPTIMAL TIME AND SPACE COMPRESSION OF TEST RESPONSES FOR VLSI DEVICES" Proc. Int. TEST CONF. 1987 pp 523-529

M. G. KARPOVSKY, P. NAGVAJARA "OPTIMAL COMPRESSORS FOR TEST RESPONSES", IEEE TRANS. ON COMPUTERS, 1989.

SIMULATION RESULTS

COMPARISON OF QUADRATIC AND LFSR COMPRESSORS PERFORMANCES.

- TWO 181-ALUs
- 20 TRIALS OF PSEUDORANDOM TEST SEQUENCES.

Circuit	No Test PATTERNS	NO FAULTS
8-bit ALU	300	240
12-bit ALU with carry look AHEAD	500	1,466

COMPARISON OF FAULT COVERAGES FOR LFSRs AND QCs

CIRCUIT	WITHOUT COMPRESSION (% undetected)	4 bit SIGNATURE		8-bit SIGNATURE	
		QC	LFSR	QC	LFSR
8-bit ALU	5.7 %	11.4%	11.8 %	5.9%	6.1%
12-bit ALU	0.7 %	6.6%	7.2 %	1.1%	1.2 %

DOES NOT DEPEND ON DISTRIBUTIONS OF ERRORS

ANALYSIS OF ALIASING PROBABILITIES OF MISRs

Let $\tilde{z}(t)$ – n-bit faulty response of a CUT at moment t.

$z(t)$ – n-bit fault-free response at moment t.

$$e(t) = (\tilde{z} \oplus z)(t) = \tilde{z}(t) \oplus z(t)$$

error at moment t.

MULTIPLICITY (WEIGHT) OF $e(t)$:

number of ts such that

$$e(t) \neq (\underbrace{0, \dots, 0}_{n})$$

ASSUMPTION: FOR ANY **GIVEN** t all nonzero $e(t)$ are **equiprobable**.
ERRORS ARE **INDEPENDENT** in TIME

Let $P(\ell)$ is a prob. of MASKING ~~detecting~~ errors with multiplicity ℓ by an r-bit primitive MISR.

then

$$P(\ell) = 2^{-r}\left(1 + (-1)^{\ell}(2^r-1)^{-\ell+1}\right)$$

EXAMPLES

$$p(1) = 0 \quad ;$$

$$p(2) = (2^r-1)^{-1} \quad ;$$

$$p(3) = \left(\left((2^r-1)^2 - (2^r-1)\right)(2^r-1)^{-3}\right. ;$$

$$p(4) = \left(\left((2^r-1)^3 - (2^r-1)^2 + (2^r-1)\right)\right)(2^r-1)^{-4}$$

etc.

FOR ANY $l > 1$

$$p(l) \approx 2^{-r}$$

for large r.

Let p is a prob that for any given t $e(t) \neq (0, \dots, 0)$ and $T < 2^r$ is a test length. Then we have for the aliasing prob.

$$P_{AL} = \sum_{l=1}^{T} \binom{T}{l} \left(\frac{p}{2^r - 1}\right)^l (1-p)^{T-l}.$$

$$2^{-r}\left((2^r-1)^l + (-1)^l (2^r - 1)\right)$$

FOR INDEPENDENT ERRORS OF EQUAL PROBABILITY (TIME INDEPENDENT SPACE SYMMETRICAL ERRORS)

FOR LARGE r

$$P_{AL} \simeq 2^{-r}$$

D.K. Pradhan, S.K. GUPTA AND M.G. KARPOVSKY "ALIASING PROBABILITY AND A NEW COMPRESSION TECHNIQUE" IEEE TRANS. ON COMPUTERS, 1990.

Conclusions

- SYNDROME TESTING, TRANSITION COUNTING AND SIGNATURE ANALYSIS ARE TIME COMPRESSION TECHNIQUES
- FOR TRANSITION COUNTING AND SIGNATURE ANALYSIS ALIASING PROBABILITY DEPENDS ON AN ORDER OF TEST PATTERNS
- PARALLEL SIGNATURE ANALYZERS ARE THE MOST POPULAR TIME COMPRESSORS.
- Syndrome testing, TRANSITION COUNTING AND SIGNATURE ANALYSIS ARE NOT ROBUST (ALIASING PROB. DEPENDS ON DISTRIBUTIONS OF ERRORS)