Random Testing

UNBIASED RANDOM TESTS

Experimental Results: (SSFs)

$$\frac{dc}{dT} = a(1-c)^b$$

c - fault coverage

T - number of test patterns

a, b - constants

b is growing with a size of DUT

Detectability Profile (for SSFs):

$$\pi = \{ \underbrace{\pi_0}_{\text{undetectable}}, \pi_1, \ldots, \pi_T \}$$

difficult to detect — easy to detect

for a given test

π_i is the number of faults detected by exactly i test patterns (number of columns in the test table with exactly i ones)

$\sum_{i=0}^{T} \pi_i = \#$ of SSFs

Example

$\pi_1 = 7$, $\pi_2 = \ldots = \pi_6 = 1$

$\pi_7 = 1$ (output S/0)

$\pi_8 = 0$

$T = 8$

Expected fault-coverage:

$$\overline{C}(T) = \sum_{i=0}^{T} \pi_i \left(1 - (1 - i/T)^T\right) \Big/ \sum_{i=0}^{T} \pi_i$$

MALAYA, YONG, Proc Int Test Conf. 1984)

MALAYA, YONG   Proc. Int. Test Conf; 1984   (2)

Faults at fanout bran[ches] ~~to~~ are differentiated

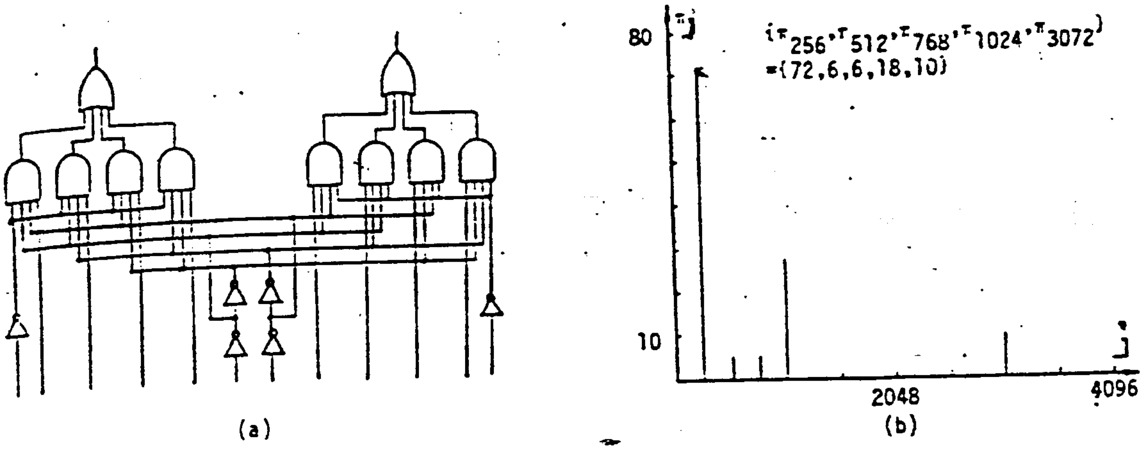


Fig.1: a. Logic diagram of a double 4-to-4 MUX(74153)
b. Detectability profile

$\sum_{i=0}^{T} \pi_i = 2N$ N - # of lines

For $\pi_i = Const \cong \frac{2N}{T}$ $\bar{C} = \frac{1}{T} \sum_{i=0}^{T} \left(1 - \left(1 - i/T\right)^{T}\right)$

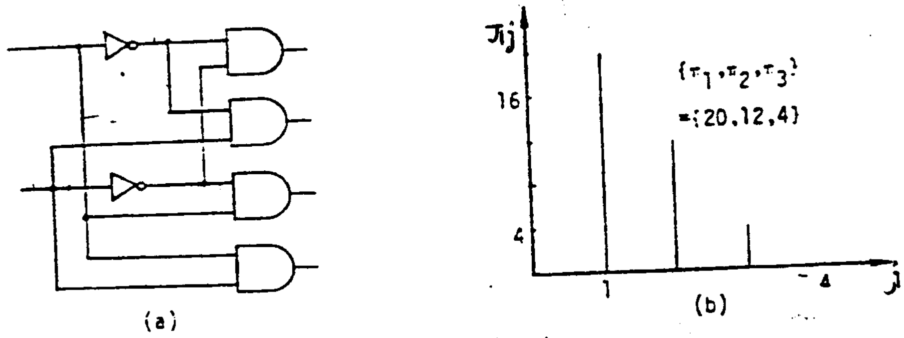


Fig.2: a. Logic diagram of a 1-of-4 decoder
b. Detectability profile

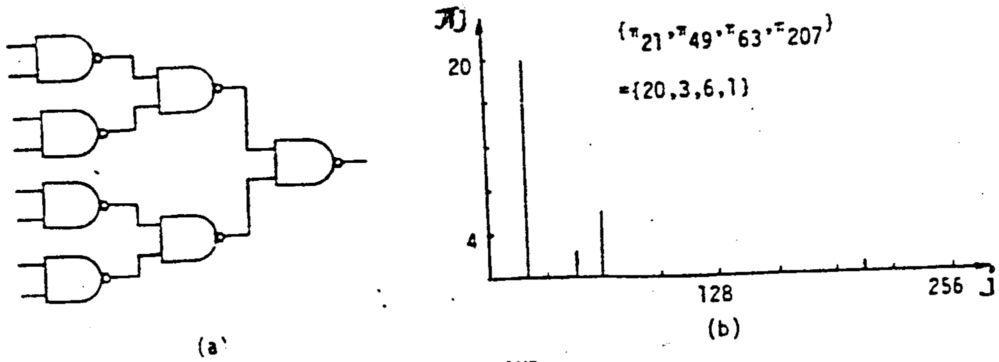


Fig.3: a. Logic diagram of a 3-level NAND-tree
b. Detectability profile

③

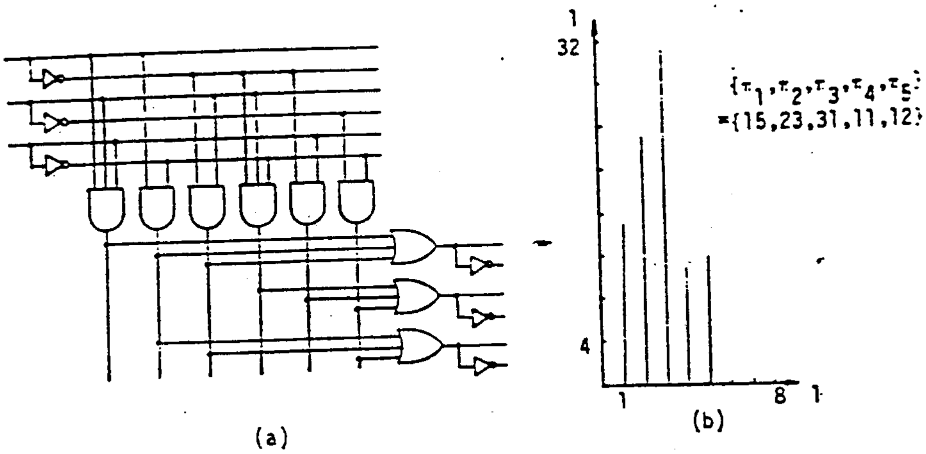


Fig.6: a. Logic diagram of a 3x6x3 PLA
b. Detectability Profile

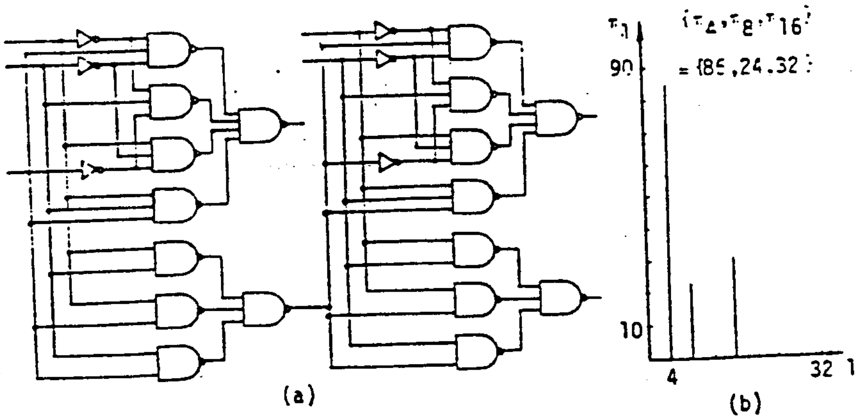


Fig.7: a. Logic diagram of a 2-stage adder
b. Detectability Profile

<u>Example 1.</u> Parity tree with m input

$Q = 2(2m-1)$ - number of faults (SSF)

$N = 2m-1$

for any fault $K = 2^{m-1}$

$$\pi_K = \begin{cases} 0, & K \neq 2^{m-1}; \\ 2(2m-1), & K = 2^{m-1}. \end{cases}$$

<u>Example 2</u> Two-stage multiplexers

(Proc. Int. Test Conf. 1987, "Probability models for pseudorandom tests" Ed McCluskey ed al)

<u>Single stuck-at-lea faults</u>

a - adress bits;

d - data bits;

$m = a + d$ - primary inputs.

$Q = 2(d(a+2)+4a+1)$ - number of faults (SS

Detectability Profile for Multiplexers

K	2^{d-2}	2^{d-1}	2^{d}	2^{m-2}	2^{m-1}
π_K	ad	$ad+3m$	$3a$	$2a$	$d+2$

easy to detect — difficul

Example 1) $a=4$, $d=16$, $m=20$, $Q=226$ to tes

K	2^{14}	2^{15}	2^{16}	2^{18}	2^{19}
π_K	64	124	12	8	18

2) $a=5$, $d=32$, $m=37$, $Q=490$

K	2^{30}	2^{31}	2^{32}	2^{35}	2^{36}
π_K	160	271	15	10	34
M	128	64	32	4	2

M - ave number of test patterns per fault.

For a probability of masking a fault with detectability K by pseudorandom test we have

$$Q_K \simeq e^{-KT/2^m}$$

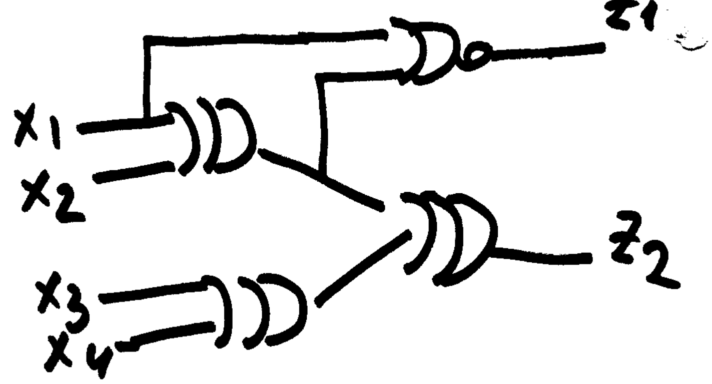
T - Length of the random test

m - number of inputs for a DUT

Fault coverage:

$$\boxed{C \geq 1 - \sum_{K} e^{-KT/2^m}}$$

EXAMPLE



$\pi_8 = 14$ → $z_1/0$ – 4 test patterns

$00**$

$\pi_4 = 1$

$\pi_{12} = 1$ → $z_1/1$ – 12 test patterns

$1***$

$01**$

If $T = 16$

$$\overline{C}(16) = 14\left(1-\left(1-8/16\right)^{16}\right) + 1\left(1-\left(1-4/16\right)^{16}\right) + 1\left(1-\left(1-12/16\right)^{16}\right)$$

$$= 14\left(1-0.5^{16}\right) + \left(1-0.75^{16}\right) + \left(1-0.25^{16}\right)$$

PROBABILITY OF A DETECTION OF A GIVEN FAULT BY A RANDOM TEST

let: α = Prob of detection of a given fault by <u>one</u> randomly chosen test pattern

P = Prob of detection of the fault by a test consisting of T randomly chosen test patterns

$$P = 1 - (1-\alpha)^T$$

For a given $\underline{P}$ one can compute test length T by

$$T = \frac{\log(1-P)}{\log(1-\alpha)}$$

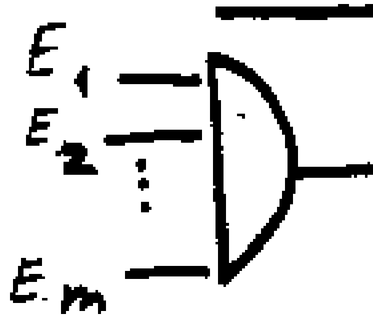
Computation of Prob. of Detection of a given fault by one random chosen test pattern (α)

(PARKER - McCluskey (PM) Algorithm

SIGNAL PROBABILITY

EXAMPLE 1.

a)

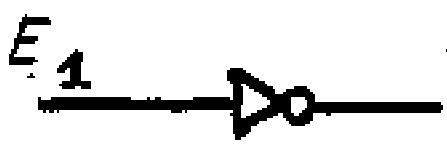


$E = E_1 \cdot E_2 \ldots E_m$

b)

$E = 1 - (1-E_1) \cdot (1-E_2) \ldots (1-E_n$

c)



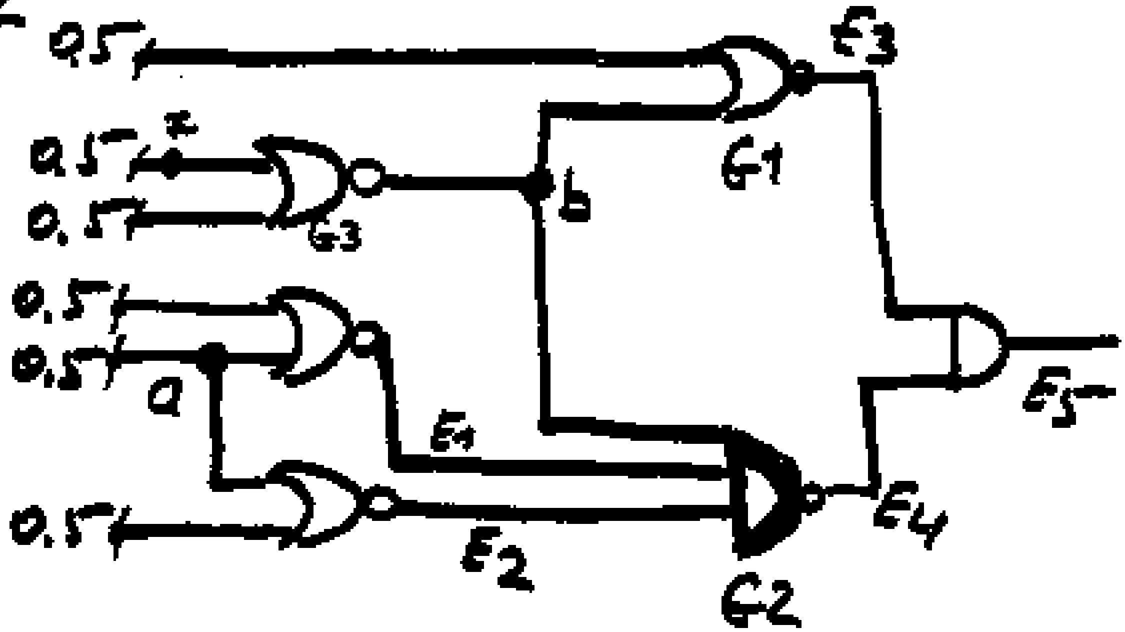
$E = 1 - E_1$

E is a prob of setting a line into 1. or of ~~detecting~~ provoking the corresponding s/0 fault.

UNBIASED RANDOM TEST with REPETITIO

1. COMPUTATION OF SIGNAL PROBABILITIES

EXAMPLE 2



$a = 0.5$, $b = 0.25$

$E_1 = E_2 = 1-(1-(1-0.5)(1-a)) = 0.5(1-a) = 0.25$

$E_3 = 1-(1-(1-0.5)(1-b)) = 0.5 \cdot 0.75 = 0.375$

$E_4 = 1 - b\,E_1 E_2 = 1 - 0.25 \cdot 0.25 \cdot 0.25 = 0.984,$

$E_5 = E_3 E_4 = 0.375 \cdot 0.984 = 0.369.$

2. COMPUTATION OF PROB. OF DETECTION BY ONE RANDOMLY CHOSEN TEST PATTERN (UPPER BOUNDS)

$\alpha(x) = (\underbrace{0.5 \cdot 0.5\, E_4}_{\text{VIA G1}} + \underbrace{0.5\, E_1 E_2 E_3}_{\text{VIA G2}})\, 0.5 -$

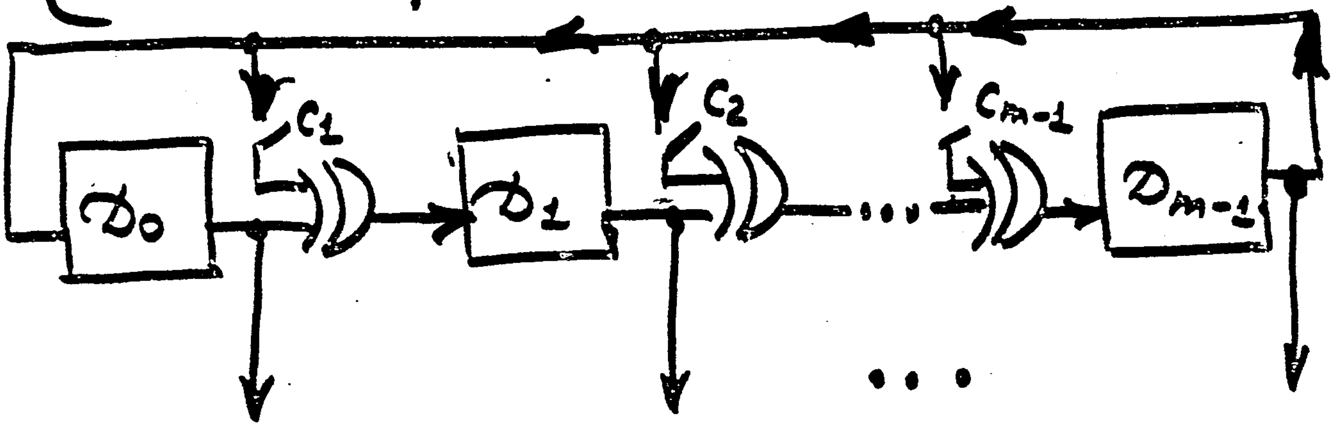
$- 0.5(\underbrace{0.5 \cdot 0.5\, E_4 \cdot 0.5\, E_1 E_2 E_3}_{\text{via both G1 and G2}}) \simeq 0.125$

J. Savir, G. S. Ditlow and H.P. Bardell "RANDOM PATTERN TESTABILITY", IEEE TRANS COMPUT C-33 (1), 79-90, 1984.

Generation of Pseudorandom Tests

Linear Feedback Shift Registers (LFSR's)

(without repetition of test patterns)

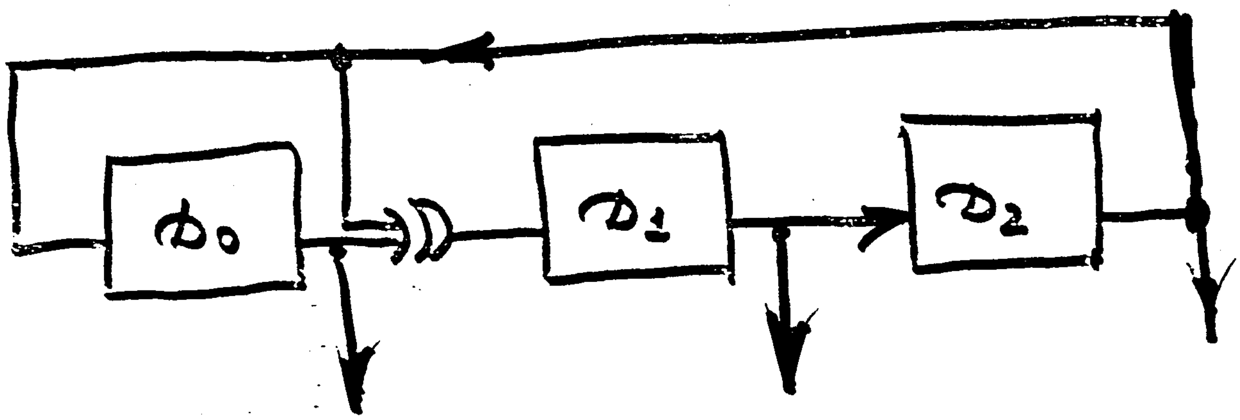


Initial state $\neq (0,\dots,0)$

Polynomial Representations of LFSR's

$$P(x) = 1 \oplus c_1 x \oplus c_2 x^2 \oplus \dots \oplus c_{m-1} x^{m-1} \oplus x^m$$

Example: $P(x) = 1 \oplus x \oplus x^3$ $(m=3)$



$K = 16, \underline{32}$

Polynomial $P(x)$ is <u>irreducible</u> if it cannot be represented as a product of two polynomials of smaller degree

$$x^2 \oplus 1 = (x \oplus 1)^2 \text{ - reducible}$$

LFSR corresponding to irreducible, primitive polynomials generate pseudorandom tests.

Ex: $X^k \oplus X \oplus 1$

Best choice ⇒ primitive trinomials

Tables of irreducible polynomials in W.W. Peterson, "Theory of Error-Correcting Codes".

Random test is a good I step in testing of <u>combinational</u> devices

[illegible] ... efficient for combinational networks
[illegible] ... to shift the [illegible] patterns.

STATISCTICAL RESISTANSE OF FAULTS

T1. FOR UNBIASED RUNDOM TEST THE AVERAGE number OF TEST PATTERNS TO DETECT NEXT FAULT is EQUAL $\frac{T+1}{K}$, where T is THE NUMBER OF TEST PATTERNS ALREADY APPLIED AND K is THE number OF FAULTS DETECTED By one TEST PATTERN of T patterns applied.

This estimation is good for large T.

Let: T_1 and T_2 be two sub-sets of RUNDOM test patterns;

N_1 and N_2 - numbers of FAULTS DETECTED BY T_1 and T_2, correspondingly;

$N_{1,2}$ - number of FAULTS DETECTED BY <u>BOTH</u> T_1 and T_2;

N_0 - total number of faults in THE DUT.

<u>T2</u>. We have FOR EXPECTED FAULT COVERAGE OF AN UNBIASED RANDOM TEST; $T_1 \cap T_2$

$$\bar{C} = \frac{N_1 N_2}{N_{12} \cdot N_0}$$