SHANNON'S THEOREM FOR BINARY CODES

For any $\varepsilon > 0$, $R < \mathcal{C}(p)$
$\exists$ $(n, 2^k)$ code $\mathcal{C}$:

$\frac{k}{n} \geq R$ and $P_{corr} \geq 1 - \varepsilon$

<u>Example</u> Since $\mathcal{C}(0.01) \simeq 0.92$

FOR $p = 0.01$ we can construct a code with rate 0.9 and $P_{corr} \geq 1 - \varepsilon$ for any ε.

ERROR DETECTION BY BINARY CODES

Let A_i is a number of codewords in $\mathbb{C}$ of weight i

$A_0 = 1$

If $d(C) = d$

$A_1 = A_2 = \dots = A_{d-1} = 0$

$A = \{A_0, A_1, \dots, A_n\}$ is <u>weight distribution</u> of $\mathbb{C}$

$$\sum_{i=0}^{n} A_i = |\mathbb{C}| = 2^k$$

PROBLEM OF COMPUTING THE WEIGHT DISTRIBUTION OF A GIVEN CODE C is difficult and still open for many codes

Denote P_{det} - prob. of error detection by code C

Then

$$\boxed{P_{det} = 1 - \sum_{i=1}^{n} A_i \, p^i (1-p)^{n-i}}$$

Example $C = \{0000, 1011, 0101, 1110\}$

$n = 4$, $k = 2$, $q = 2$

$A_0 = 1$; $A_1 = 0$, $A_2 = 1$, $A_3 = 2$

$$P_{det} = 1 - p^2(1-p)^2 - 2p^3(1-p)$$

FOR $p = 0.01$ ~~$P_{det} = 0.9999$~~

$P_{det} = 0.9999$

One out of 10,000 errors will not be detected by this code