PRODUCTS OF LINEAR CODES

MULTIDIMENSIONAL CODES

Let V_1 is (n_1, q^{k_1}, d_1) q-ary code

V_2 is (n_2, q^{k_2}, d_2) q-ary code

Consider: $V = V_1 \times V_2$

codewords are $n_1 \times n_2$

q-ary matrices

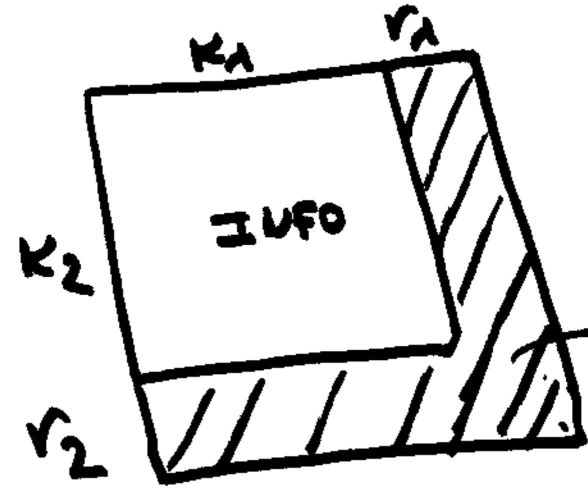
information bits are $k_1 \times k_2$

submatrices of codewords

Every row in a codeword for

V is a codeword of V_1 and every

column is a codeword of V_2



$n_1 = k_1 + r_1$

$n_2 = k_2 + r_2$

redund. bits.

T. $V = V_1 \times V_2$ is the

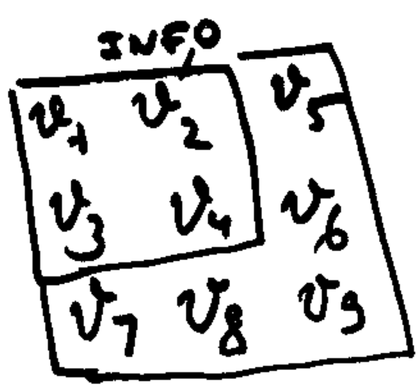
$(n_1 \cdot n_2, q^{k_1 \cdot k_2}, d_1 \cdot d_2)$ code

<u>Example</u>

2-dim. parity

$V_1 = V_2$ $(n, 2^{n-1}, 2)$ parity code

$n = 3$



Redundand symbols are defined as

$v_5 = v_1 + v_2$, $v_6 = v_3 + v_4$ (row checks)

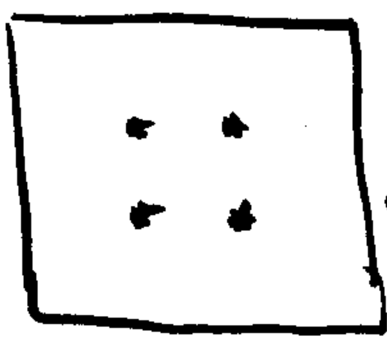
$v_7 = v_1 + v_3$, $v_8 = v_2 + v_4$ (column checks)

$v_9 = v_5 + v_6 = v_1 + v_2 + v_3 + v_4$

$= v_7 + v_8$

2 dim parity is the code with $n = s^2$, $k = (s-1)^2$, $d = \underline{4}$

The only errors which are not detected by 2-dim parity are



$\Leftarrow$ in the same two columns and in the same two rows

3-d PARITY.

DATA is organized as a 3-d cube
PARITY CHECK FOR ANY one OF
THE 3 dimensions

$V = V_1 \times V_1 \times V_1 = V_1^3$ where

$V_1 = (s, 2^{s-1}, 2)$ 1-dim parity

For $V = V_1^3$ $\quad n = s^3$, $k = (s-1)^3$

$d = 8.$

For t-d PARITY $\quad (t = 1, 2, \ldots)$

$V = V_1^t$

$n = s^t, \; k = (s-1)^t, \; d = 2^t$

COMPARISON OF 2-d. PARITY with EXTENDED HAMMING CODES

$d=4$

SHORTENED

n	r_{2d}	r_{HAMM}
4	3	3
9	5	5
16	7	5
25	9	6
36	11	7

HAMMING CODES ARE MORE EFFICIENT THAN 2-d PARITY

PRODUCTS OF HAMMING CODES and one dim parity codes

$$V = V_1 \times V_2 \qquad V_1 \quad \left(n_1,\ 2^{n_1 - \lceil \log_2(n_1+1) \rceil},\ 3\right)$$

HAMMING

$$\underline{d=6} \qquad V_2 \quad \left(n_2,\ 2^{n_2-1},\ 2\right)$$

PARITY

$$V = \left(n_1 \cdot n_2,\ 2^{n_1 - \lceil \log_2(n_1+1) \rceil + n_2 - 1},\ 6\right)$$

EXAMPLE

$$n_1 = 5 \Rightarrow k_1 = 2 \qquad \left(\lceil \log_2(5+1) \rceil = 3\right)$$

$$H_1 = \left[\begin{array}{cc|ccc} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right], \quad G_1 = \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$V_1 = \begin{array}{cc} 00 & 000 \\ 01 & 101 \\ 10 & 011 \\ 11 & 110 \end{array} \qquad \text{HAMMING} \quad d_1 = 3$$

$$n_2 = 3, \quad k_2 = 2, \quad d_2 = 2$$

$$n = n_1 \cdot n_2 = 15, \quad k = k_1 \cdot k_2 = 4, \quad d = d_1 \cdot d_2 = 6$$

$V_1 \times V_2$

LIST OF CODEWORDS:

00|000 00|000 00000	00|000 01|101 01101	00|000 10|011 10 011
00|000 11|110 11 110	01|101 00|000 01 101	01|101 01|101 00 000
01|101 10|011 11 110	01|101 11|110 10 011	10|011 00|000 10 011
10|011 01|101 11 110	10|011 10|011 00 000	10|011 11|110 01 101
11|110 00|000 11 110	11|110 01|101 10 011	11|110 10|011 01 101
11|110 11|110 00000		