# Single Error Correcting Codes in Lee Metric

Error patterns

$$(\underbrace{0, \dots 0, \pm 1, 0, \dots, 0}_{n})$$

Thus for single error-correction, if $H = [h_1, h_2, \dots, h_n]$ is a check matrix then

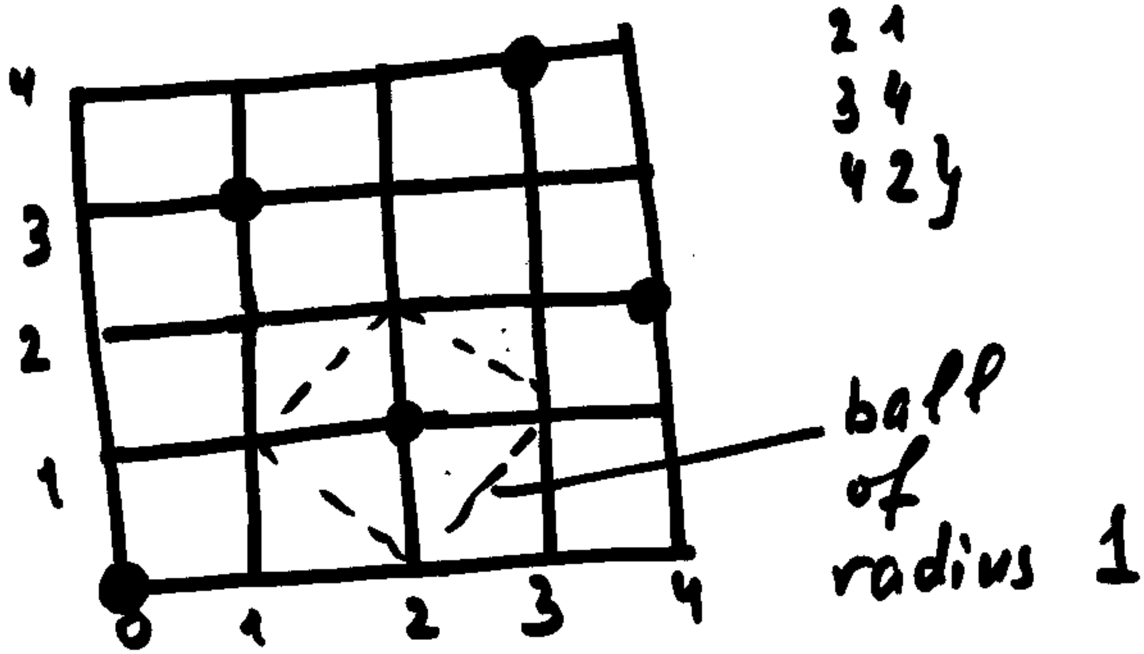
$$c_1 h_i + c_2 h_j \neq 0$$

$$c_1, c_2 \in \{0, \pm 1\}$$

**Example** $n=2$, $q=5$

$H = [2, 1]$ $\quad$ $P = [3]$

$G = [1, 3]$ $\quad$ $V = \{00, 13, 21, 34, 42\}$




This code is *perfect*

$(2,5,3)$ code over $\mathbb{Z}_5$

If $q \geq 2n+1$ then

$$H = [1 \ \ 2 \ \ 3 \ \ \ldots \ \ n-1 \ \ n \ ]$$

$v \in V$ iff

$$v_1 + 2v_2 + 3v_3 + \ldots + (n-1)v_{n-1} +$$

$$+ n v_n = 0 \pmod{q}$$

(Similar to ISBN code but for ISBN $n=10$, $q=11$)

Thus, for $q \geq 2n+1$ we have $(n, q^{n-1}, 3)$ Lee codes with $r = n-k = 1$ and rate $1 - \frac{1}{n}$

For $q = 2n+1$ these $(n, q^{n-1}, 3)$ Lee codes are <u>perfect</u>

Since $V(1)$ volume of a ball of radius 1 in Lee metric is $2n+1$

and for any single-error correcting code $V$

$$|V| = q^k \leq \frac{q^n}{V(1)} = \frac{q^n}{2n+1}$$